Exam ~~1~~ X3
Chapters ~~2 and 3~~ 4, 5 and 6

Answer the following questions. *Answers without proper evidence of knowledge will not be given credit.* Make sure to make reasonable simplifications. Do not approximate answers. Give exact answers. **No calculators are allowed on this exam.**

True or False (2 points each)

T **1.** Two angles are coterminal if they differ by a multiple of 2π radians.

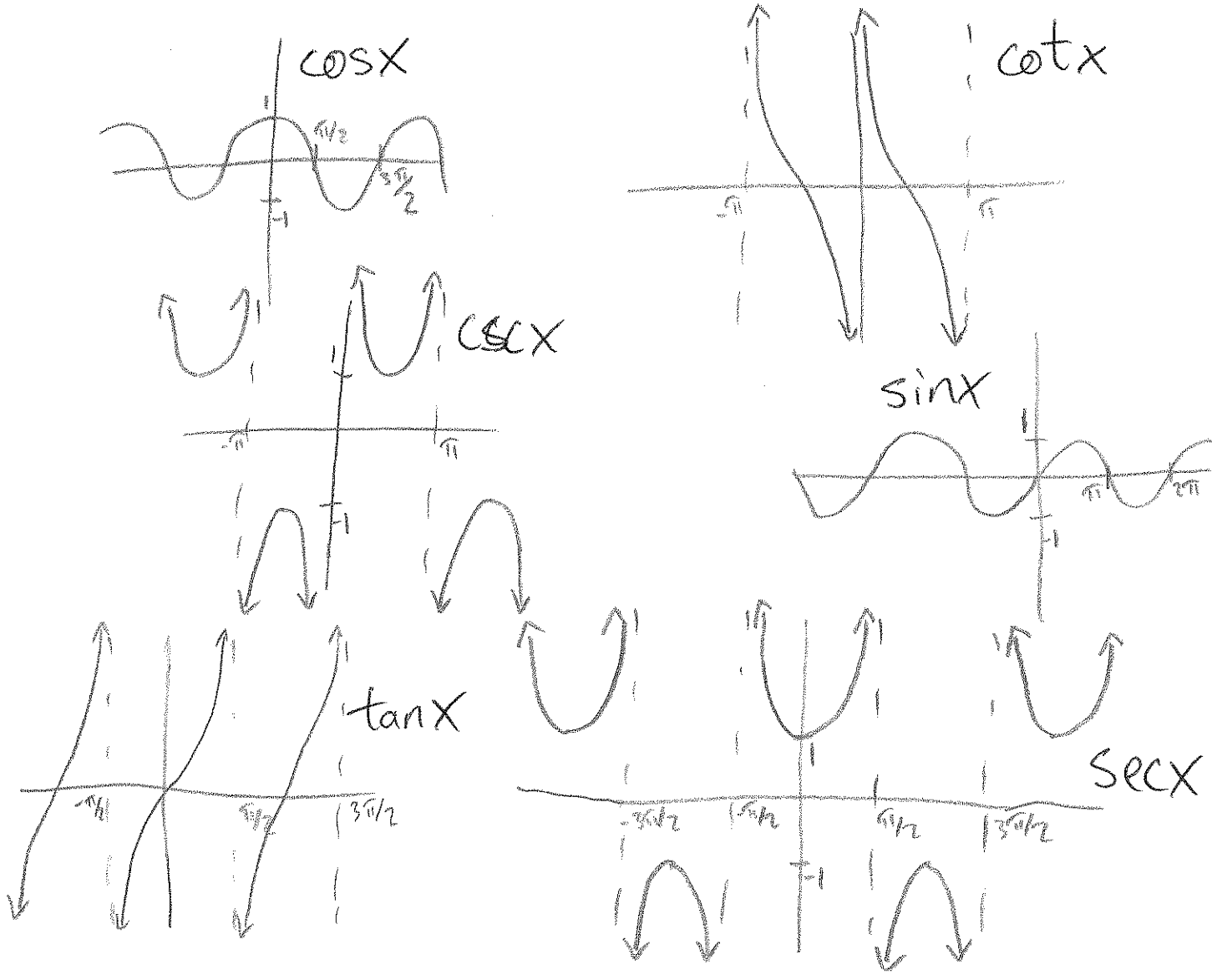
T **2.** $\sin x = \cos(\pi/2 - x)$

F **3.** $ln(MN) = ln(M) \cdot ln(N)$

T **4.** $e^{x+y} = e^x \cdot e^y$

Matching Graphs

Label each graph below with the appropriate trigonometric function. (2 points each)



Show your work!

1. Solve the following equations.(3 points each)

Correction!

$\log_3(27)=Y$

a) ~~$\log_3(y) = 27$~~

b) ~~$\ln(0) = x$~~

c) $4^x = 1/16$

$\ln(1)=X$

a) $\log_3(27)=Y \iff 3^Y=27 \Rightarrow Y=3$

b) $\ln(1)=x \iff e^x=1 \Rightarrow x=0$

c) $4^x=1/16 \Rightarrow x=-2$

2. Evaluate the following functions. Remember the range of the inverse trigonometric functions. (3 points each)

a) $\tan(3\pi/4)$

b) $\arcsin(-\sqrt{3}/2)$

c) $\cos(14\pi/3)$

a) $\tan\left(\frac{3\pi}{4}\right)=\frac{\sin(3\pi/4)}{\cos(3\pi/4)}=\frac{\sqrt{2}/2}{-\sqrt{2}/2}=-1$

b) $\arcsin\left(-\sqrt{3}/2\right)=x \iff \sin x=-\sqrt{3}/2$

$x=-60^\circ, -\pi/3$ radians

c) $\cos(14\pi/3)=\cos(2\pi/3)=-\frac{1}{2}$

3. Evaluate the following functions. (3 points each)

a) $\arccos(1/2)$

b) $\arcsin(\cos(2\pi/3))$

c) $\sec(\arcsin(1/\sqrt{2}))$

d) $\tan(2\arcsin(-1/2))$

a) $\arccos(1/2) = x \iff \cos x = 1/2$

$x = \pi/3$ radians, $60°$

b) $\arcsin(\cos(2\pi/3)) = \arcsin(-1/2)$

$= -30°, -\pi/6$ radians

c) $\sec(\arcsin(1/\sqrt{2})) = \sec(\arcsin(\sqrt{2}/2))$

$= \sec(\pi/4)$

$= \frac{1}{\cos(\pi/4)}$

$= \frac{1}{\sqrt{2}/2} = \sqrt{2}.$

d) $\tan(2\arcsin(-1/2)) = \tan(-\pi/3)$

$= \frac{\sin(-\pi/3)}{\cos(-\pi/3)}$

$= \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$

4. Use the sum,difference, double-angle and/or half-angle formulas to evaluate **two** of the following. Clearly indicate which two you would like graded. (10 points each)

a) $\cos(7\pi/12)$

b) $\tan(3\pi/8)$

c) $\sin(285°)\cos(15°) - \cos(285°)\sin(15°)$

a) $\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{7\pi/6}{2}\right) = \pm\sqrt{\frac{1+\cos(7\pi/6)}{2}}$

$= \pm\sqrt{\frac{1+-\sqrt{3}/2}{2}}$

$= \pm\sqrt{\frac{2-\sqrt{3}}{4}} = \pm\frac{\sqrt{2-\sqrt{3}}}{2}$

$\frac{7\pi}{12}$ in Quadrant II so

$\cos\left(\frac{7\pi}{12}\right) = -\frac{\sqrt{2-\sqrt{3}}}{2}$

b) $\tan\left(\frac{3\pi}{8}\right) = \tan\left(\frac{3\pi/4}{2}\right) = \frac{\sin(3\pi/4)}{1+\cos\left(\frac{3\pi}{4}\right)}$

$= \frac{+\sqrt{2}/2}{1+-\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{2-\sqrt{2}} \cdot \frac{2+\sqrt{2}}{2+\sqrt{2}}$

$= \frac{2\sqrt{2}+2}{4-2} = \frac{\sqrt{2}+1}{2}$

c) $\sin(285°)\cos(15°) - \cos(285°)\sin(15°)$

$\sin(285°-15°) = \sin(270°) = -1$

5. Determine if the following pairs of angle are coterminal. (5 points each)

a) $\alpha = 750°$ and $\beta = -30°$

b) $\gamma = 5\pi/2$ and $\theta = -3\pi/2$

a) $750° - (-30°) = 780°$

$2(360°) = 720°$ so NO.

b) $\frac{5\pi}{2} - \left(-\frac{3\pi}{2}\right) = \frac{8\pi}{2} = 4\pi = 2(2\pi)$

So Yes.

6. Use the sum-to-product and product-to-sum identities to evaluate each of the following functions. (5 points each)

a) $\cos(5\pi/24)\sin(-\pi/24)$

b) $\sin(285°) - \sin(15°)$

a) $\cos\left(\frac{5\pi}{24}\right)\sin\left(\frac{-\pi}{24}\right) = \frac{1}{2}\left[\sin\left(\frac{5\pi}{24} - \frac{\pi}{24}\right) - \sin\left(\frac{5\pi}{24} - \left(\frac{-\pi}{24}\right)\right)\right]$

$= \frac{1}{2}\left[\sin\frac{\pi}{6} - \sin\frac{\pi}{4}\right]$

$= \frac{1}{2}\left[\frac{1}{2} - \frac{\sqrt{2}}{2}\right] = \frac{1-\sqrt{2}}{4}$

b) $\sin(285°) - \sin(15°) = 2\cos\left(\frac{285°+15°}{2}\right)\sin\left(\frac{285°-15°}{2}\right)$

$= 2\cos(150°)\sin(135°)$

$= 2\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{6}}{2}$

7. Verify the following identities.

a) $1 + \sec x \sin x \tan x = \sec^2 x$ (3 points)

b) $\frac{\sec x}{\tan x} - \frac{\tan x}{\sec x} = \cos x \cot x$ (7 points)

a) $1+\sec x \sin x \tan x = 1 + \frac{1}{\cos x} \cdot \sin x \cdot \frac{\sin x}{\cos x}$

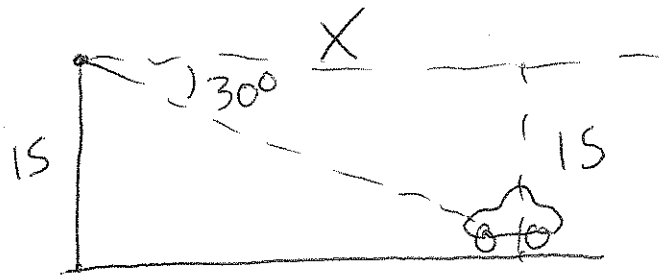
$= 1 + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2 x = \sec^2 x.$

b) $\frac{\sec x}{\tan x} - \frac{\tan x}{\sec x} = \frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}} - \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \frac{1}{\sin x} - \frac{\sin x}{1}$

$= \frac{1-\sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x} = \cos x \cdot \frac{\cos x}{\sin x} = \cos x \cot x$

Extra Credit

EC 1. From a highway overpass, 15 meters above the road, the angle of depression of an oncoming car is measured at 30°. How far is the car from the point on the highway directly below the observer? (5 points)



$\tan(30°) = \frac{15}{x}$

$\tan(30°) = \frac{\sin(30°)}{\cos(30°)} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

So $\frac{\sqrt{3}}{3} = \frac{15}{x} \Rightarrow x\sqrt{3} = 45 \Rightarrow x = \frac{45}{\sqrt{3}} = \frac{45\sqrt{3}}{3}$

$= 15\sqrt{3}$ m

EC 2. Solve for x in the equation $\log_5(x) - \log_5(x-2) = 3$.

$3 = \log_5(x) - \log_5(x-2) = \log_5\left(\frac{x}{x-2}\right)$

So

$5^3 = \frac{x}{x-2}$

$125 = \frac{x}{x-2}$

$125(x-2) = x$

$125x - 250 = x$

$125x - x = 250$

$124x = 250$

$x = \frac{250}{124} = \frac{125}{62}$.